Dimensional analysis.

A. Introduction

This handout was originally written for another purpose. It was intended as a self-paced review for students to use on their own, with staff available for help. It was part of a set of several such handouts. Some of the other handouts are posted at the web site, and some may be referred to in class handouts as "supplementary materials" that are available.

Dimensional analysis is a problem solving tool. It is particularly useful as a guide in solving problems that might be difficult to do otherwise. To emphasize learning the method -- the tool -- we introduce dimensional analysis with some easy examples. You could undoubtedly do many of these problems in your head. The point for now is to learn how to use the tool and to see that it works. The key to dimensional analysis is following the units.

Many of the examples and problems in this handout use "common" English (USCS) units, for familiarity. (Most chem books should contain a list of conversion factors for these, including metric-English equivalents. See, for example Table 3.3. of Cracolice. If I use one you don't know, please ask.) Metric problems are also included. If you need help with the metric units, see Sect 3.4 of Cracolice.

Within the problem sets, some problems are marked with an *. This indicates that the problem introduces something new. If you are skipping around in the problems, you may well want to stop for a bit at a problem marked with an *.

I did not intend significant figures (SF) to be an issue when I wrote the original version of this. However, some who use this now may care about SF. The answers do now show the correct number of SF. Exception: In some simple problems, with simple integer data, I have treated the data as "exact".

B. Simple unit conversions; the idea of dimensional analysis

Example. Convert 2 feet to inches.

There is a relationship between the two units in this problem, the "given" (feet) and the "wanted" (inches). 12 in = 1 ft. This simple equation can also be written

$$\frac{12\ \text{in}}{1\ \text{ft}} = 1 \quad \text{or} \quad \frac{1\ \text{ft}}{12\ \text{in}} = 1$$

You know that multiplying a number by 1 does not change its value. So multiplying by 12 in/1 ft, which equals 1, does not change the value. But it does change the form, the units; in fact, this is precisely the goal.

$$2\ \cancel{\text{ft}} \times \frac{12\ \text{in}}{1\ \cancel{\text{ft}}} = 24\ \text{in}$$

In writing out this dimensional analysis solution, we start by writing the "given" (2 ft). We then write an appropriate conversion factor. What is appropriate?

– First, it must be true; that is, the factor must equal 1.

– Second, it must be useful in the problem, relating the given to the wanted. (In this case, there is a simple, single step relationship. We will see more complex cases later.)

– Third, we write the conversion factor in a useful way. The conversion factor would be just as true if we had written it upside down. But we wrote it so that the ft cancel out, and the desired unit (inches) is left, in the numerator.

> [For clarity, when working a problem always write conversion factors vertically, as in the above example. In ordinary writing (and typing) we often write conversion factors (and complex units) on a single line. Example: 12 in/ft. However, when solving problems we need to be able to easily tell the numerator from the denominator; this is easier when all fractions are written vertically.]

A major strength of dimensional analysis is in guiding you how to write the conversion factor. You just write the conversion factor so that "it works" -- the appropriate units cancel or remain. If you write a conversion factor upside down, it won't work. (We will show an example of this in Section D, below.) You see that, and turn it over. The units guide you to a correct solution.

Problems. Use dimensional analysis in solving the following problems. Emphasize showing clear work, even if you could do some of the simple problems in your head. Using dimensional analysis on such simple problems helps you to see that the method works. If you learn the method with simple problems, you will be able to apply it later to more complex ones.

I suggest that you turn in some of your work for this handout (each section). Certainly, show me problems that still bother you. But I would also like to look at how you do (set-up) at least some of the problems. Dimensional analysis is, in part, a matter of good form.

1. Convert 48 inches to feet. 2. Convert 5 days to hours.
3. How many seconds are there in 4 minutes?
4. How many quarts are in 10 gallons? 5. How many ounces are in 5.20 pounds?

6. How many meters are in 24 millimeters?
7. Convert 27.2 cm to meters.
8. Convert 0.76 kg to grams.
9. Convert 3.50 liters to mL.
10. Convert 1500 mg to g.

C. Multi-step conversions

Example. How many ounces are there in 3 gallons?

This is equivalent to asking: Convert 3 gallons to ounces. Hm. How are ounces and gallons related? Offhand, I don't know (and let's assume you don't either). But they are both units of volume, and both are related to quarts. That is, I don't see a one-step relationship between the given and the wanted, but I do see a two-step relationship. Gallons → quarts → ounces. (Some authors refer to this as the "unit path".)

Using dimensional analysis…

$$3\ \cancel{\text{gal}} \times \frac{4\ \cancel{\text{qt}}}{\cancel{\text{gal}}} \times \frac{32\ \text{oz}}{\cancel{\text{qt}}} = 384\ \text{oz}$$

In this case we use two conversion factors, but that doesn't change the basic idea. In fact, a useful feature of the method is that it is just as easy to use when many factors are involved as when only one is needed.

⇒ Note that we showed clearly which units cancel, and therefore which is/are left.

The key points are that -- for each factor…

– The factor has a value of 1. That is, the conversion factor is true.

– The factor is written in a way to help the units work out correctly. If one way seems wrong, turn the factor upside down, and try again. The reciprocal of 1 is 1.

Why not just use 128 oz/gal, and do the problem in one step?? That's fine -- if you really do know that there are 128 ounces in a gallon. Very few people do know that. And if you have to do a preliminary calculation to find that, you haven't gained anything.

Again, the method for converting units used here doesn't make any special demands. You can use any (correct) conversion factors and any number of them. *Nothing is gained by trying shortcuts.* If you are comfortable with a shortcut, ok, but it's not worth spending time hunting for one. Just do one step at a time, and make sure each step is correct.

If you find a problem difficult because you can't figure out how to get from "here" (the given) to "there" (the wanted), focus on finding a unit path. Identify the given and wanted, and then think about how they are related. Explore some possibilities, if you don't see an obvious or simple relationship. Get the units to work before worrying about the numbers. In fact, you might even set up the whole problem with just units, no numbers, to see the structure. Then put in the numbers and do the calculations -- after knowing that you have the structure of the solution.

> How about showing a two step conversion by writing out two separate one-step conversions? In some sense it doesn't matter, if it works out ok. But there are two advantages to showing one large conversion, showing all the steps in one big set-up:
>
> - Showing one large set-up lets you check that the units work for the entire problem before you do any calculations.
> - If you show steps separately and round off at each step, then you reduce the quality of the answer by accumulating rounding errors. It is better to do only one big calculation, and round only once -- at the end.

Problems. Most of these problems are designed to be done with two (or more) conversion factors. Of course, you might be able to do some in one step, by knowing a more complex factor. That's ok, but remember that nothing is gained if you have to do a side calculation to figure out an "obscure" conversion factor. I encourage you to use simple factors in these problems, to see the ease of using two or more factors in a single problem.

This set of problems also raises a couple of other issues, which haven't been introduced above. There are brief comments about them in the answer section. See me if you need more help.

11. Convert 1 Ms to hr.
12. Convert 8.7 cm to mm.
13. Convert 2500 mg to kg.
* 14. Convert 147 cm/s to m/s.
* 15. Convert 60 miles/hour to feet/second.
16. Convert 8.6 g/mL to pounds/gallon.
17. Convert 43 mg/dL to g/L.
18. Convert 4.7 g/L to mg/mL.
19. Convert 1 Ångstrom to picometers. (1 Å = 10^{-8} cm)
20. How many milliliters are in one Tablespoon? (A Tbl is 1/64 of a quart.)
* 21. Consider a square with area = 0.13 m^2. Convert that area to cm^2.
22. How many liters are in a cubic meter?
23. A typical car may get 30 miles per gallon of gasoline. What is this in metric units, km/L?
24. When the space shuttle is moved around on the ground, it is carried on a truck at about 1 mile per hour. What speed is this in metric units, m/s?

D. Velocity and density as conversion factors

Most of the preceding problems were "simple" conversions from one unit to another unit for the same property (e.g., length → length). Dimensional analysis is also useful in solving problems that might be considered algebraic.

Velocity is distance per unit time (for example, mi/hr or m/s). Velocity, then, can be thought of as a conversion factor between distance and time. Similarly, density (lb/gal or g/mL) is a conversion factor between mass and volume.

You can do problems involving velocity or density using "well known" algebraic equations -- if you know them. Or, you can use the units inherent in the problem to guide you to the correct relationship. Of course, velocity and density are only two examples of a large class of such proportionality relationships. You will soon be using a chemical proportionality, between grams and moles. Dimensional analysis will guide you to proper use of this relationship, even while you are not entirely sure what a mole is.

The following example illustrates the use of dimensional analysis to guide you through a velocity problem.

Example. How long does it take to travel 100 miles at 50 miles/hour?

One approach is to remember a formula (distance = rate x time, or d=rt), then solve it for time: t=d/r. This is fine -- if you remember the formula correctly *and* if you do the algebra correctly.

But suppose you're not sure of the formula. You may vaguely recall that d, r, and t are related, but you're not sure how. Lost? Not at all. Just follow the units. You know that a mile is a unit of distance, and so forth. The problem gives you a distance and a rate (or velocity). The problem asks for a time. Your answer must come out in time units. How? Experiment a little, and you will see that

$$\text{hours} = \cancel{\text{miles}} \times \frac{\text{hours}}{\cancel{\text{miles}}}$$

or, going back to properties…

$$\text{time} = \cancel{\text{distance}} \times \frac{\text{time}}{\cancel{\text{distance}}}$$

or, in symbols…

$$t = d * (1/r) = d/r$$

Now that's the same formula we had before. Of course -- because it is right. The difference is that earlier we relied on having memorized the formula, whereas this time we figured it out -- from the units.

> Further, if we had used the first approach to find that t=d/r, checking the units would serve as a check that we were correct.

Putting in the numbers…

$$t = 100\ \cancel{\text{mi}} \times \frac{\text{hr}}{50\ \cancel{\text{mi}}} = 2\ \text{hr}$$

To emphasize the usefulness of carefully following the units, we show an incorrect solution to the problem. Suppose that, for some reason, your memory or algebra told you that t=dr. Putting in the numbers, 100 x 50 = 5000. And you know the answer is in hours. So you turn in an answer of 5000 hr. That may sound silly in this example, but it reflects a common kind of error.

Let's do this more carefully…

(???) $t = dr$

(???) $$t = 100 \text{ mi} \times \frac{50 \text{ mi}}{\text{hr}} = \frac{5000 \text{ mi}^2}{\text{hr}}$$

Look at the units. You know the answer must be in time units (hours) -- BUT IT ISN'T. That means something is wrong. Go back and look over the problem, to find out what.

It serves no purpose to just write "hours" because you know it has to be "hours."

⇒ There is a separate supplementary worksheet that focuses on density. (Some density problems are included below, but see the Density worksheet for more attention to this topic.)

Problems

25. How long would it take an airplane traveling at the speed of sound (740 miles per hour; "mach 1") to travel around the earth, at the equator? The circumference of the earth is 25,000 miles.
26. What speed would be needed for an airplane to travel around the earth (again, at the equator) in 10 hr?
27. We are currently moving toward Los Angeles at the rate of 0.02 m/year (slipping along the San Andreas fault). At this rate, how long will it take us to reach L.A., 640 km away?
28. How much does 15.0 cm^3 of aluminum weigh? The density of aluminum is 2.70 g/cm^3.
29. What is the volume of 25 g of ethyl alcohol (density = 0.79 g/mL)?
* 30. The relationship between electric power (P), current (i) and voltage (V) is given by P=iV. i is in amps, V is in volts; therefore P is in amp·volts. A more common unit of power is the watt: 1 watt = 1 amp·volt. Ordinary house current is 120 volts. How many amps of current are drawn by a 60 watt light bulb?
31. An electric heater is listed as drawing a current of 10 amps. What is its power, in kilowatts?

E. Answers

In doing these problems, the emphasis is as much on showing clear work as on getting the right answer. The answer key may alert you if you have made a mistake. However, I would also like to see your work for this handout; I will comment on how you apply dimensional analysis as well as on your answers. (I have shown a few set-ups here.)

Sect B

1. $48\ \cancel{\text{inches}} \times \dfrac{1\ \text{ft}}{12\ \cancel{\text{inches}}} = 4.0\ \text{ft}$

2. $5\ \cancel{\text{days}} \times \dfrac{24\ \text{hr}}{\cancel{\text{day}}} = 120\ \text{hr}$

3. 240 s	4. 40 qt	5. 83.2 oz	6. 0.024 m
7. 0.272 m	8. 760 g	9. 3500 mL	10. 1.5 g

Sect C

11. Overall path: Ms→hr. Suggested path: Ms→s→min→hr.

$$1\ \cancel{\text{Ms}} \times \frac{10^6\ \cancel{\text{s}}}{1\ \cancel{\text{Ms}}} \times \frac{\cancel{\text{min}}}{60\ \cancel{\text{s}}} \times \frac{\text{hr}}{60\ \cancel{\text{min}}} = 278\ \text{hr}$$

12. 87 mm 13. 0.0025 kg 14. 1.47 m/s (With complex units, do one thing at a time. In this case, convert cm to m, and then s to s. Hm.)
15. 88 ft/s (One thing at a time, remember?) 16. 72 lb/gal
17. 0.43 g/L 18. 4.7 mg/mL 19. 100 pm 20. 15 mL
21. 1300 cm^2 (Did you get 13 cm^2??? You used 100 cm/m as a conversion factor? Yes, but… You have m^2 in this problem.)
22. 1000 23. 13 km/L 24. 0.45 m/s

Sect D

25. Miles (given) → time (no specific unit was specified, hours will be convenient)

$$25{,}000\ \cancel{\text{mi}} \times \frac{\text{hr}}{740\ \cancel{\text{mi}}} = 34\ \text{hr}$$

26. 2500 mi/hr	27. $3.2\text{x}10^7$ yr	28. 40.5 g
29. 32 mL	30. 0.5 amp	31. 1.2 kw