Solutions: Dilutions.



This handout was originally written for another purpose. It was intended as a self-paced review for students to use on their own, with staff available for help. It was part of a set of several such handouts. Some of the other handouts are mentioned here; if you are interested in one of them, let me know.

This handout follows Molarity. One problem uses material from Percentage. However, for the most part, Dilutions and Percentage are independent; they can be done in parallel, or in either order.

## A. Dilutions: Introduction

In the previous work we discussed preparing solutions from the pure components (for example, solid NaCl solute and the solvent, water). However, sometimes you prepare one solution from another. In particular, it is common to take a concentrated solution and prepare a dilute solution, by adding more solvent. (The terms concentrated and dilute are qualitative terms that refer to a -- relatively -- high or low concentration, respectively, of solute in the solution.)

Preparing solutions by dilution is convenient. One can make a single concentrated stock solution of a solute, and then quickly prepare a variety of more dilute solutions by dilution. (Why is it easier? Measuring volumes is quicker than weighing.) Further, dilution of a concentrated solution is a practical way to make solutions that are so dilute you would have trouble weighing the solute.

There is a simple equation for calculating dilutions. It is also easy to present the logic of the equation.

We mainly discuss dilution problems using molarity as the concentration unit.

Within the problem sets, some problems are marked with an *. This indicates that the problem introduces something new. If you are skipping around in the problems, you may well want to stop for a bit at a problem marked with an *.

I did not intend significant figures (SF) to be an issue when I wrote the original version of this. However, some who use this now may care about SF. The answers do now show the correct number of SF. Exception: In some simple problems, with simple integer data, I have treated the data as "exact".

## B. The dilution equation

$V_c M_c = V_d M_d$.

In this equation, V = volume and M = molarity. The subscripts c and d refer to the concentrated and dilute solutions, respectively.

### Example

Need: 100 mL of 1.0 M NaCl. Have: a 5.0 M NaCl stock solution.

To make the desired solution, we will take some volume ($V_c$) of the concentrated solution, and then add water (solvent) until the total volume is the desired 100 mL. The question is, how much of the concentrated solution do we need, or what is $V_c$?

Given: $V_d$ = 100 mL (= 0.10 L); $M_d$ = 1.0 M; $M_c$ = 5.0 M. If you can identify these three givens, then you can simply use the dilution equation to calculate $V_c$. So as you do this example -- and real problems -- emphasize identifying the givens. In common dilution problems, you know the desired volume and concentration of the new, dilute solution ($V_d$ and $M_d$), and you know the concentration of the stock solution ($M_c$); the goal is to find out how much of the stock you need ($V_c$).

$V_c M_c = V_d M_d$

$$V_c = \frac{V_d M_d}{M_c} = \frac{0.10\ \text{L} \times 1.0\ \cancel{\text{M}}}{5.0\ \cancel{\text{M}}} = 0.020\ \text{L}\ (= 20\ \text{mL})$$

Thus, $V_c$ = 20 mL. To make the desired solution, take 20 mL of the 5.0 M stock solution, then add enough water to make 100 mL total volume of solution. (As discussed before, the denominator of molarity is volume of solution; the final volume is determined by measuring the solution, not the amount of solvent added.)

Note about the unit M. In this problem we wrote M in the problem, and we canceled M's. That may seem to contradict previous advice that you should (usually) expand M in problems, to mol/L. No harm would be done by expanding M here, but that isn't needed. How do you know? Look at the units in the problem. In this case, we do not need to deal with the mol or L separately; the M's simply cancel. (Previous M problems dealt with converting moles $\rightleftarrows$ L.)

Note about $V_c$, and a hidden assumption. $V_d$ is simple enough; it is the amount of the dilute solution you are making. It may be tempting to think that $V_c$ is the amount of the concentrated solution you have. WRONG. It is the amount you use. So how much do you have? Don't know, except that it is plenty (an "excess"). You must have "enough" of the concentrated to do what you want, but the amount you have per se is not an issue. You calculate how much you need, and take that from the stock. (If there isn't enough stock, someone needs to make more; see Molarity.)

Pitfall; important check. The most common mistake in doing dilution calculations is to use the equation backwards, to get c and d at least partially mixed up. The first protection against this problem is to be careful with the units. Look at the example above. The M units cancel, and the only remaining unit is the volume unit -- in the numerator. Thus the set-up gives volume, which is what we want. If your set-up doesn't give volume, or gives volume in the denominator, it is wrong. Watch the units. (Remember, the point of using dimensional analysis is to guide you toward correct set-ups and away from incorrect ones. Just writing the unit you know you want, even if it doesn't come out of the set-up, seriously misses the value of using dimensional analysis.)

Unfortunately, dimensional analysis alone won't protect you from the second part of the pitfall -- but common sense will. What if you mix up the two concentrations, putting the wrong one on top? You will still get volume, but you will get the wrong volume -- an obviously unreasonable volume. In the example, if you reverse the two concentration terms, you will get $V_d$ = 500 mL. This "answer" says to take 500 mL of stock solution, and add water until you get to 100 mL. Nonsense! Obviously nonsense!

The overall message… watch the units and check that the result seems reasonable. These precautions are good general precautions in all problems.

If you can't exactly recall the dilution equation, the precautions may be enough to guide you to do the calculation correctly. The desired volume must be the given volume times the ratio of concentrations; that will give you volume. And the ratio of concentrations must be written so that the answer comes out as reasonable.

## C. The logic of the dilution equation

> [You do not need to understand the basis of the dilution equation in order to properly use it. However, the equation is rather easy to understand, and understanding it can help develop your intuition about how to approach problems. Further, if you do dilution calculations only occasionally, it may actually be easier and safer to solve them "from first principles" than to try to remember the dilution equation and to use it properly.]

Think moles. Once again, the mole underlies chemical logic.

You know that

$$\text{moles} = \text{volume} * \text{molarity} = \cancel{\text{volume}} * \frac{\text{moles}}{\cancel{\text{volume}}}$$

(where volume means volume of solution).

Recall the Example in Sect B, above. The desired solution is 100 mL ($V_d$) of 1.0 M NaCl ($M_d$). Using the above relationship, we <u>need</u> $V_dM_d$ = 0.10 L * (1.0 mole/L) = 0.10 mole of NaCl.

If we were making the solution from solid solute, we would convert the amount needed (0.10 mole) to mass, using the molar mass. (See the Molarity handout.)

But in this case we don't need to weigh out the NaCl. We already have some NaCl in solution, in a 5.0 M ($M_c$) solution. How can we get the 0.10 mole we need from a solution that contains 5.0 M = 5.0 mole/L? Well, that 0.10 mole must also = $V_cM_c$. Rearranging the relationship, $V_c$ = moles/$M_c$ = 0.10 mole/(5.0 mole/L) = 0.020 L. (Write that out clearly, so you can see that the units work.)

This result, 0.020 L = 20 mL, is what we calculated earlier from the dilution equation. That equation combines the two steps: how many moles of solute do we need; what volume of stock does it take to get that many moles.

## D. Should you "memorize" the dilution equation? -- Attention X11 students

In this worksheet, I started by presenting the dilution equation, in Sect B. I then described the logic of that equation, in Sect C. In Sect B, I cautioned about making sure you are clear what the terms means, especially the subscripts c and d. And at the start of Sect C, I cautioned that if you do dilutions only occasionally, you might actually be better off to not use the dilution equation.

This worksheet was originally written for a review class, in which most of the students were trying to become more comfortable with chemical calculations. Learning to use the dilution equation, which presumably they had learned about previously, seems reasonable in this context.

However, some students are now using this worksheet as a supplement for an introductory chemistry class, and focusing on the dilution equation may not be the best approach. In fact, I minimize discussion of the dilution equation in the introductory course. Dilutions can be easily and logically done "from first principles", as discussed in Sect C:

1. How many moles of solute do you need?
2. Where do you get those moles from (in this case, what volume of the concentrated solution)?

It is so easy to solve dilution problems that way that I really encourage it for beginning students. Those who use dilutions a lot will undoubtedly want to remember how to use the dilution equation, since it is a little bit quicker. But it is not much quicker, and for occasional use really may not be worth the risk of using it incorrectly.

It is ok for students in my introductory classes to use the dilution equation, but I give no partial credit for using it incorrectly. If you are not sure how to use it, use the logical two-step procedure of Sect C.

## E. Practical notes

Most common dilutions tend to involve “round numbers”, such as diluting 2-fold, 10-fold, 50-fold. This results from “good planning” (??) and common usage of round number concentrations.

Sometimes it is desirable to do a dilution in two steps. For example, if calculation tells you to do a dilution using 0.01 mL of the stock… Well, that’s hard to measure. To achieve the goal, we could do the dilution in two steps. Each step would satisfy the dilution equation, and after the second step, we would have the desired final concentration. The choice of intermediate step is more or less arbitrary; your judgment and preference for round numbers will guide you. More about doing multiple dilution steps in Sect I

What is the limit on what you can do in a single step? That depends on the measuring devices available to you, and the required precision. If you are using ordinary serological pipets, volumes below 0.1 mL are not recommended for high precision work. (Most of the problems below can be done with one step.)

How concentrated can/should stock solutions be? “Can” is easy. It can be anything -- up to the solubility limit (which you can look up in a handbook). Should? That’s mainly convenience, and relates in part to the previous point. If you have some cases you would like to discuss, please see me.

In doing dilution calculations, $V_d$ is the volume of the final, dilute solution. We add enough water (solvent) to reach this volume, just as we did in making “molar” solutions from the solid (Molarity handout). However, we often “cheat” a little. In most dilution situations, volumes are very nearly additive. Thus it works reasonably to add a measured volume of water, such that $V_c$ + measured water = $V_d$. If you do this, remember that it is an approximation, although very often an adequate one.

## F. Dilutions involving other concentration units

If the concentration units have “volume of solution” (= total volume) as the denominator (the same as for molarity), then the general logic of the dilution equation still holds. One of the problems illustrates this, using %(w/v). (See the Percentage handout for general work with that unit.)

If the concentration units do not have “volume of solution” as the denominator, then you cannot do simple dilution calculations by volume. What you need to do depends on the specific units, and is beyond our general discussion here. (There is some discussion of interconverting concentration units in the Percentage handout.)

## G. Problems

1. Given a 4.00 M stock of sodium chloride, NaCl, how would you prepare 100 mL of 1.00 M NaCl?
2. How would you make 5.00 L of 0.200 M NaCl from the same stock solution?
3. You have a 1.0 M stock of "Tris" (a common buffer). You want 10 L of 0.10 M Tris. Procedure?
4. How would you make 50 mL of 0.010 M Tris from the above stock?
5. You need 100 mL of 0.040 M potassium dihydrogen phosphate, $KH_2PO_4$. How much of a 0.50 M stock solution would you use?
* 6. Consider the same situation as the previous problem. But instead of just using the dilution equation, let's start from the beginning, and do it one step at a time. Using $V_d$ and $M_d$, how many moles of $KH_2PO_4$ do you want in the final solution?
7 . Continuing… How many moles of $KH_2PO_4$ will you take from the stock?
8. Continuing… What volume of the stock do you need to get those moles?
9. Given a 2.5 M stock, how would you make 500 mL of 0.022 M potassium chloride, KCl?
10. How would you make 12 L of 0.15 M sodium nitrate, $NaNO_3$, given a 3.0 M stock?
* 11. You have a 1,000 L tank, and want the medium in it to contain 3.0 mM magnesium chloride, $MgCl_2$. How much of a 1.0 M stock solution would you add?
* 12. You make a sodium bromide solution by diluting a 2.5 M stock solution as follows: 100 mL of the stock is added to enough water to bring the total volume to 1.0 L. What is the final NaBr concentration?
13 . You dilute 500 mL of a 0.40 M stock of magnesium acetate, $Mg(C_2H_3O_2)_2$ to 4.0 L. What is the concentration of the diluted solution?
14 . Given a 1.0 mM stock, how would you make 10 mL of 10 μM ATP?
15 . Continuing… How would you make 1.0 mL of a 10 μM solution? Assume that you can do the required measurements in one step.
* 16. Continuing… How would you proceed if the volume (previous question) is considered too small to measure?
* 17. Continuing… What is the mass of ATP in this solution (1 mL of 10 μM)? (Molar mass = 507 g/mol.) Would it be practical to make the desired solution by directly weighing out the solute?
* 18. Given a 0.50 M stock solution of NaCl, how would you make 250 mL of a 2.0 M NaCl solution?
* 19. You often use 0.020 M NaCl, 0.10 M NaCl and 0.20 M NaCl, usually 10-100 mL at a time. For convenience, you want to have a single stock solution that can be used to make all of these as needed. Suggest a desirable stock solution. Tell how you would make it; include amounts. And tell how you would make <u>one</u> of the desired dilute solutions.
* 20. You want 10.0 L of a growth medium to contain 0.20% glucose (w/v). You have a 40%(w/v) stock solution of glucose. How much of the stock solution do you need?
* 21. You need 2.0 L of a solution containing 0.20 M NaCl, 0.40 M $KNO_3$, 1.0 mM $CaCl_2$, 10 mM Tris buffer. How would you make this? The following solutions are available: 10 mM $CaCl_2$; 1.0 M Tris.

## H. Dilution factor

"Dilution factor" is a useful term. It allows you to talk about a dilution without referring to the specific volumes or concentrations that are involved. It describes what the dilution accomplishes.

The dilution factor (DF) is the ratio of the old (c) and new (d) concentrations. Equivalently, it is the ratio of the new and old volumes. Recall the dilution equation, Sect B.

$$DF = \frac{M_c}{M_d} = \frac{V_d}{V_c}$$

### Example

Using the example of Sect B, where a 5.0 M stock was diluted to 1.0 M, DF = 5.0 M/1.0 M = 5. Or, the ratio of volumes: 100 mL/20 mL = 5. The DF is a unitless number, as the ratio of two numbers with the same units.

I have expressed the DF as a number >1, e.g., 5 in this case. Some people may do it the other way, with DF = 1/5 in this case. It doesn't really matter. If you understand the logic of dilution, it will be obvious what the DF means.

Another way to refer to a DF is to say that a dilution is X-fold, for example 5-fold in the example. This terminology was used in Sect E.

Problems. (By convention, all DF are given as >1.)

22-26. Calculate the DF for problems #1-5, above.
27. You have a 2.5 M solution of cobalt chloride, $CoCl_2$, and you dilute it by a factor of 10. What is the concentration of the diluted solution?

## I. Multiple dilutions; serial dilutions

What if you do one dilution, then dilute that dilution? What is the total dilution factor? The issue of multiple dilutions was introduced very briefly in Sect E and problem #16; let's look at it further.

Some people are surprised by the answer, so I think it is useful for you to figure it out. Let's consider an example. So you can discover the answer yourself, I'll pose the example as a series of problems.

Problems

28. You have a 15 M solution of potassium hydroxide, KOH. You dilute it 5-fold. What is the new concentration?
29. You now dilute the dilution you made above 3-fold. What is the new concentration?
30. What single DF could have been used instead of the two used together in the two previous questions?

With luck, you now see that DF's combine by multiplication. Each dilution alone involves a multiplication (= division) operation; successive dilutions then are successive multiplications.

Review problem #16. The goal is to dilute 100-fold (data given in #15). You can do that using one 100-fold dilution. Or you can do it in 2 steps, 10-fold each. Or you can do a 5-fold dilution followed by a 20-fold dilution. Or you could do a 4-fold dilution followed by a 20-fold dilution followed by a 1.25-fold dilution. In each case, the total DF -- the product of the individual DF's -- is 100, which is what you want. All of these procedures are correct; some are simpler, some may be more appropriate for certain situations.

Serial dilutions. This term is often used for multiple dilutions, especially when multiple dilutions with the same DF are done. (I'm not sure that is a restriction of the term.) For example, a microbiologist may do serial 10-fold dilutions of a bacterial culture. If you do three successive (serial) 10-fold dilutions, the total DF = 10 x 10 x 10 = $10^3$. That is… If you do N X-fold dilutions, the total DF is $X^N$.

Problems

31. You do a 10-fold dilution, followed by a 5-fold dilution. What is the total DF?
32. You have 12 M hydrochloric acid, HCl. You do two consecutive 10-fold dilutions. What is the final HCl concentration?
33. You do 3 100-fold serial dilutions of a bacterial culture. What is the total DF?
34. You do serial 2-fold dilutions of a drug sample. What is the total DF for the 6th dilution tube?

## J. Answers

1. Need 25.0 mL of the 4.00 M stock solution. Then add water to reach 100 mL total volume. In "chemical shorthand", dilute 25.0 mL of the 4.00 M stock to 100 mL.
2. Dilute 0.250 L of the 4.00 M stock to 5.00 L.
3. Dilute 1.0 L of the 1.0 M stock to 10 L.
4. Dilute 0.50 mL of the 1.0 M stock to 50 mL.
5. 8.0 mL 6. 0.0040 mol
7. 0.0040 mol (Same as previous question. That's the point. See Sect C.)
8. 8.0 mL (Same as #5. What you have done in these last three problems is equivalent to what you did in #5. In that case, you did it in one step, using the dilution equation, Sect B. Then you essentially worked it out from first principles, as explained in Sect C.)

9. Dilute 4.4 mL of the 2.5 M stock to 500 mL.
10. Dilute 0.60 L of the 3.0 M stock to 12 L.
11. 3.0 L (mM means "millimolar" = millimoles/L)
12. 0.25 M 13. 0.050 M
14. Dilute 0.10 mL of stock to 10 mL.
15. Dilute 10 µL of stock to 1.0 mL.
16. The total dilution is 100-fold (1.0 mM/10 µM). This could be done in two successive 10-fold steps. Take 0.10 mL of stock, dilute to 1.0 mL; take 0.10 mL of that, dilute to 1.0 mL. Try putting these numbers through the dilution equation. (Other strategies would also work; I will be happy to look at yours.) More discussion of multiple dilutions is in Sect I.
17. 5.1 µg. Not practical to weigh. Of course, you could make a large volume of the solution, and store the rest for other needs. But making and storing a small volume of a more concentrated solution is easier.
18. By dilution, you can't.
19. There are many possible and reasonable answers. For example, you might make a 1.0 M stock solution -- which is more concentrated than any of the solutions you want to use. To make 1.0 L of this, you would weigh out 58.5 g of NaCl(s), and add enough water to make the total volume 1.0 L. To make 0.020 M NaCl from this, you would dilute 50-fold; to make 50 mL of it, take 1.0 mL of the 1.0 M stock, add water to 50 mL final volume. Please have me check your answer.
20. 0.050 L (= 50 mL). The denominator of %(w/v) is volume of solution, just as it is for molarity. Thus the general logic of dilution is the same. (See Sect F.)
21. Weigh out 23.4 g of NaCl and 80.9 g of $KNO_3$. Measure 0.20 L of the $CaCl_2$ stock and 20 mL of the Tris stock. Mix all these together, and add enough water to bring <u>total</u> volume to the desired 2.0 L. (As a practical matter, one might dissolve the solids each in a small amount of water, before mixing the solutes together. Sometimes, it is easier to dissolve one chemical at a time. That doesn't affect the overall logic or the calculations. The <u>total</u> volume is 2.0 L.) It's common to need a solution containing several solutes -- for example, for growth media or enzyme reactions. And so you need to do lots of calculations, but each one is independent. The particular example here is from my imagination, and I tried to keep the numbers fairly simple, but this is an example of a practical put-it-all-together problem.

| | | | |
|---|---|---|---|
| 22. 4 | 23. 20 | 24. 10 | |
| 25. 100 | 26. 12.5 | 27. 0.25 M | |
| 28. 3.0 M | 29. 1.0 M | 30. 15 | |
| 31. 50 | 32. 0.12 M | 33. $10^6$ | 34. $2^6 = 64$ |

dil
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